AQA

A-LEVEL
MATHEMATICS

Mechanics 5 – MM05
Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	Candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1	$a\omega = 1.3$	B1		Award B1 for $a^2\omega^2 = 1.3^2$ OE.
	$1.2^2 = \omega^2(a^2 - 0.2^2)$ $1.2^2 = \left(\frac{1.3}{a}\right)^2 (a^2 - 0.2^2)$ $1.44 = 1.69 - \frac{0.0676}{a^2}$ $a^2(1.69 - 1.44) = 0.0676$	M1A1		M1: Equation with correct terms, but may contain sign errors. A1: Correct equation. dM1: Solving for *a*. A1: Correct *a*. A1: Correct *AB*
	$a = \sqrt{\frac{0.0676}{(1.69 - 1.44)}} = 0.52$	dM1A1		Accept $\frac{26}{25}$
	$AB = 2 \times 0.52 = 1.04\,\text{m}$	A1	6	
	Total		**6**	

Q	Solution	Mark	Total	Comment
2(a)	$ml\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -mg\sin\theta$	M1A1		M1: Equation of motion with correct terms. A1: Equation with correct terms and signs.
	No air resistance / $\sin\theta \approx \theta$	B1		B1: Correct assumption. (Allow sphere is a particle.)
	$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{mg\theta}{ml}$ $\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g}{l}\theta$ **AG**	A1	4	A1: Correct simplification to obtain final answer.
b(i)	$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{9.8}{0.392}\theta = -25\theta$	B1		B1: Obtaining 25θ
	$\theta = A\sin(5t) + B\cos(5t)$	M1		M1: Expression for θ with two unknown constants. May include their values instead of 5.
	$t = 0, \theta = \frac{\pi}{10} \Rightarrow B = \frac{\pi}{10}$	A1		A1: Correct value for *B*
	$t = 0, \dot{\theta} = 0 \Rightarrow A = 0$ $\theta = \frac{\pi}{10}\cos(5t)$	A1	4	A1: Correct value for *A* and correct final answer.
b(ii)	$\frac{\pi}{15} = \frac{\pi}{10}\cos(5t) \Rightarrow \text{t} = 0.16821$ $\frac{\pi}{30} = \frac{\pi}{10}\cos(5\text{t}) \Rightarrow \text{t} = 0.24619$ $0.24619 - 0.16821 = 0.0780$	M1A1 dM1 A1	4	M1: Forming two equations using $\frac{\pi}{15}$ and $\frac{\pi}{30}$ A1: Correct equations dM1: Obtaining two solutions. A1: Correct final answer. Accept 0.078
	Total		**12**	

Q	Solution	Mark	Total	Comment
3(a)	$r=3$	B1	1	B1: Correct value.
(b)	$r\dot{\theta}=5$ $r^2\dot{\theta}=\text{Constant}=r\times r\dot{\theta}=3\times5=15$ $\dot{\theta}=\frac{15}{r^2}=\frac{15}{9}(1+\sin\theta)^2$ $=\frac{5}{3}(1+\sin\theta)^2$ **AG**	B1 M1 dM1 A1	4	B1: Statement of $r\dot{\theta}=5$ M1: Use of $r^2\dot{\theta}=\text{Constant}$ dM1: Solving for $\dot{\theta}$ A1: Correct expression for $\dot{\theta}$ from correct working
c(i)	$\dot{r}=-3(1+\sin\theta)^{-2}\cos\theta\dot{\theta}$ $=-5\cos\theta$	M1 A1	2	M1: Differentiating *r* wrt *t* A1: Correct expression for $\dot{r}$
c(ii)	$\ddot{r}=5\sin\theta\dot{\theta}=\frac{75\sin\theta}{r^2}$ $\ddot{r}-r\dot{\theta}^2=\frac{75\sin\theta}{r^2}-r\times\frac{225}{r^4}$ $=\frac{75\sin\theta}{r^2}-\frac{225}{r^3}$ $=\frac{1}{r^2}(75\sin\theta-225\frac{(1+\sin\theta)}{3})$ $=-\frac{75}{r^2}$ $k=-75$	M1A1 dM1 A1	4	M1: Differentiating $\dot{r}$ wrt *t* A1: Correct expression for $\ddot{r}$. dM1: Applying $\ddot{r}-r\dot{\theta}^2$ A1: Correct final answer with correct value of *k*.
	or $\ddot{r}=5\sin\theta\dot{\theta}=\frac{75\sin\theta}{r^2}$ $\ddot{r}-r\dot{\theta}^2=\frac{25}{3}\sin\theta(1+\sin\theta)^2-\frac{3}{(1+\sin\theta)}\left(\frac{25}{9}\right)(1+\sin\theta)^4$ $=\frac{25}{3}\left(\frac{3}{r}-1\right)\left(\frac{3}{r}\right)^2-\frac{75}{9}\left(\frac{3}{r}\right)^3$ $=-\frac{75}{r^2}$ $k=-75$	(M1A1) (M1) (A1)		
	Total		**11**	

Q	Solution	Mark	Total	Comment
4(a)	$m\dfrac{d^2x}{dt^2}=mg-T$	M1		M1: Use of Newton's second Law with weight and tension.
	$=mg-\dfrac{mg}{0.2}\left(x-0.2-0.1\sin(4t)\right)$	M1A1		M1: Expression for the tension with three terms for the extension. A1: Correct expression for tension.
	$=mg(1-5x+1+0.5\sin(4t))$			
	$\dfrac{d^2x}{dt^2}=-5gx+2g+0.5g\sin(4t)$	M1		M1: Rearranged to the required format.
	$\dfrac{d^2x}{dt^2}+49x=19.6+4.9\sin(4t)$ **AG**	A1	5	A1: Correct result from correct working.
(b)	**CF** $x=D\sin(7t)+E\cos(7t)$	B1		B1: Correct CF with two unknown constants.
	PI $x=A+B\sin(4t)+C\cos(4t)$	M1		M1: Correct form of PI
	$\dot{x}=4B\cos(4t)-4C\sin(4t)$			dM1: Correct derivatives of PI
	$\ddot{x}=-16B\sin(4t)-16C\cos(4t)$	dM1		
	$49A+33B\sin(4t)+33C\sin(4t)$ $=19.6+4.9\sin(4t)$			B1: Correct constant term.
	$A=\dfrac{19.6}{49}=0.4$	B1		
	$B=\dfrac{4.9}{33}=\dfrac{49}{330}$			M1: Attempting to find B and C.
	$C=0$	M1A1		A1: B and C correct.
	$x=0.4+\dfrac{49}{330}\sin(4t)$			
	$x=D\sin(7t)+E\cos(7t)+0.4+\dfrac{49}{330}\sin(4t)$			M1: Using initial position to find E. A1: Correct E.
	$x=0.4, t=0$	M1		
	$E=0$	A1		
	$v=7D\cos(7t)+\dfrac{196}{330}\cos(4t)$	M1		M1: Using initial velocity to find D. A1: Correct D.
	$v=0, t=0$			
	$D=-\dfrac{28}{330}$			
	$x=-\dfrac{28}{330}\sin(7t)+0.4+\dfrac{49}{330}\sin(4t)$	A1	10	
	Total		**15**	

Q	Solution	Mark	Total	Comment
5(a)	$(m+\delta m)(v+\delta v)+(-\delta m)(v-U)-mv=-mg\delta t$ $mv+m\delta v+v\delta m+-v\delta m+U\delta m-mv=-mg\delta t$ $m\delta v+U\delta m=-mg\delta t$ $m\frac{\delta v}{\delta t}+U\frac{\delta m}{\delta t}=-mg$ $m\frac{dv}{dt}+U\frac{dm}{dt}=-mg$ $\frac{dv}{dt}=-\frac{U}{m}\frac{dm}{dt}-g$ **AG**	M1A1 dM1 A1	4	M1: Correct terms, with possible sign errors. A1: Correct equation. dM1: Simplification. A1: Correct result from correct working.
(b)	But : $m=M-\lambda t$ and $\frac{dm}{dt}=-\lambda$ $\frac{dv}{dt}=\frac{\lambda U}{M-\lambda t}-g$ **AG**	M1 A1	2	M1: Use of expressions for m and $\frac{dm}{dt}$
(c)	$v=\int\left(\frac{\lambda U}{M-\lambda t}-g\right)dt$ $=-U\ln(M-\lambda t)-gt+c$ $v=0,t=0\Rightarrow c=U\ln(M)$ $v=U\ln(M)-U\ln(M-\lambda t)-gt$ $=U\ln\left(\frac{M}{M-\lambda t}\right)-gt$	M1A1 M1A1 A1	5	M1: Integration to give ± correct ln term. A1: Correct integration M1: Finding constant of integration. A1: Correct constant A1: Correct expression
(d)	$M-\lambda t=\frac{M}{5}$ $t=\frac{4M}{5\lambda}$ $v=U\ln\left(\frac{M}{M-\frac{4M}{5}}\right)-\frac{4Mg}{5\lambda}$ $=U\ln 5-\frac{4Mg}{5\lambda}$	M1 A1 M1 A1	4	M1: Equation for time with $\frac{M}{5}$ A1: Correct time M1: Finding v A1: Correct v. Accept $1.61U-0.8\frac{Mg}{\lambda}$ OE
	Total		**15**	

Q	Solution	Mark	Total	Comment
6. (a)	$EPE = \frac{4mg}{2a}\left(2a\sin\left(\frac{\theta}{2}\right) - a\right)^2$	M1A1		M1: Attempt at EPE A1: Correct EPE
	$GPE = \frac{mga}{2}\cos\theta$	B1		B1: Correct GPE.
	$V = \frac{mga}{2}\left(16\sin^2\left(\frac{\theta}{2}\right) - 16\sin\left(\frac{\theta}{2}\right) + 4 + \cos\theta\right)$ $= \frac{mga}{2}\left(8(1-\cos\theta) - 16\sin\left(\frac{\theta}{2}\right) + 4 + \cos\theta\right)$	dM1		dM1: Finding total and simplifying.
	$= \frac{mga}{2}\left(12 - 7\cos\theta - 16\sin\left(\frac{\theta}{2}\right)\right)$ **AG**	A1	5	A1: Correct final answer from correct working.
(b)	String must be taut for expression to be valid. (As the natural length of the string is equal to the radius an equilateral triangle will be formed when the string is just taut and so the inequality must hold and is needed on both sides of the vertical.)	B1	1	B1: Mentioning that the string must be taut.
(c)	$\frac{dV}{d\theta} = 0$ $0 = 7\sin\theta - 8\cos\left(\frac{\theta}{2}\right)$	M1A1 dM1		M1: Differentiation. A1: Correct derivative dM1: Setting derivative equal to zero.
	$0 = 14\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) - 8\cos\left(\frac{\theta}{2}\right)$ $0 = \cos\left(\frac{\theta}{2}\right)\left(14\sin\left(\frac{\theta}{2}\right) - 8\right)$ $\cos\left(\frac{\theta}{2}\right) = 0$ or $\sin\left(\frac{\theta}{2}\right) = \frac{4}{7}$	dM1		dM1: Solving for θ.
	$\theta = \pi$ or 1.22 or 5.07	A1A1	6	A1: One correct solution. A1: Two other correct solutions
(d)	$\frac{d^2V}{d\theta^2} = \frac{mga}{2}\left(7\cos\theta + 4\sin\left(\frac{\theta}{2}\right)\right)$	M1		M1: Correct second derivative.
	$\theta = 1.22 \quad \frac{d^2V}{d\theta^2} = (+4.7...)\frac{mga}{2} \therefore$ Stable	A1		A1: One correct explanation and conclusion.
	$\theta = \pi \quad \frac{d^2V}{d\theta^2} = (-3)\frac{mga}{2} \therefore$ Unstable	B1		B1: Correct explanation and conclusion for $\theta = \pi$.
	$\theta = 5.07 \quad \frac{d^2V}{d\theta^2} = (+4.7...)\frac{mga}{2} \therefore$ Stable	A1	4	A1: Correct explanation and conclusion for third solution.
	Total		**16**	
	TOTAL		**75**	